Technical note

Penetration of electric fields into a concentric-sphere model of biological tissue

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1 Introduction

THE INTERACTION of externally applied electric fields with living organisms has become an important subject, and there have been reports describing effects in animals (MARINO *et al.*, 1977), clinical applications (BASSETT *et al.*, 1982), and possible public-health implications (BECKER and MARINO, 1982). The fields can act only if physically present in or on tissue, and knowledge of their magnitude and distribution must be correlated with observed physiological responses before a deeper understanding of the interaction process itself can emerge. The structural complexity of even the simplest living systems, however, has prevented direct measurement of field distributions in tissue in most reported experiments.

Mathematical modelling of the interaction process is an alternative approach to the study of electric-field penetration into tissue (HART and MARINO, 1976; MARCHESI and PARODI, 1982). In modelling, both the geometry and structural organisation of actual tissue must be significantly simplified to permit a practical calculation. The absence of clearly established tissue electrical properties is another limitation on the modelling approach. Nevertheless, modelling is the best method presently available for relating the actual tissue fields produced in the various biological studies.

In this paper we present an exact calculation based on a mathematical model comprising a sphere of tissue surrounded by a concentric shell of a second tissue. The concentric structure, and a third medium in which it is embedded, can have arbitrary relative permittivities and conductivities. The calculation yields the steady-state electric field in each tissue following application of an electric field of arbitrary magnitude and frequency in the outer medium. The general behaviour of the result is described, and we show how it can be applied to specific studies including human and animal studies, effects on tissues and cellular effects. A preliminary description of some of this work has already been given (MARINO *et al.*, 1980).

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2 Mathematical model

Consider a sphere of radius a surrounded by a shell of radius b, embedded in a third medium (Fig. 1). Assume that the electric field in each region is:

$$\begin{aligned} \boldsymbol{E}_1 &= \boldsymbol{E}_1' \cos\omega t + \boldsymbol{E}_1'' \sin\omega t \\ \boldsymbol{E}_2 &= \boldsymbol{E}_2' \cos\omega t + \boldsymbol{E}_2'' \sin\omega t \\ \boldsymbol{E}_3 &= \boldsymbol{E}_3' \cos\omega t + \boldsymbol{E}_3'' \sin\omega t \end{aligned}$$

Far from the sphere $\boldsymbol{E}_1 \rightarrow (E_0' \cos\omega t + E_0'' \sin\omega t)\boldsymbol{l}_z$, where $\boldsymbol{l}_z$ is the unit vector along the z-axis.

The potential in the three regions can be written as follows:

$$\begin{aligned} V_1 &= [\sum A_n' r^{-(n+1)} P_n(\cos\theta) - E_0' r \cos\theta] \cos\omega t \\ &\quad + [\sum A_n'' r^{-(n+1)} P_n(\cos\theta) - E_0'' r \cos\theta] \sin\omega t \\ V_2 &= \sum [B_n' r^{-(n+1)} + C_n' r^n] P_n(\cos\theta) \cos\omega t \\ &\quad + \sum [B_n'' r^{-(n+1)} + C_n'' r^n] P_n(\cos\theta) \sin\omega t \\ V_3 &= \sum F_n' r^n P_n(\cos\theta) \cos\omega t \\ &\quad + \sum F_n'' r^n P_n(\cos\theta) \sin\omega t \end{aligned}$$

where $P_n(\cos\theta)$ are the Legendre polynomials.

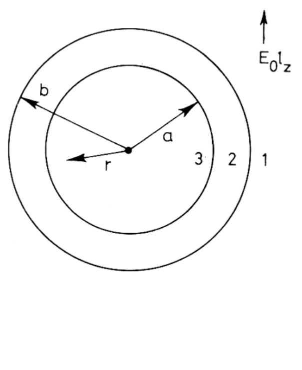

Fig. 1 *Sphere surrounded by a concentric shell and immersed in a third medium*

The eight sets of constants must be evaluated by use of the boundary conditions. It can be shown that the constants are identically zero for all values of n except $n = 1$. This fact significantly simplifies equations for the constants. At $r = b$, $V_1 = V_2$, and at $r = a$, $V_2 = V_3$. Imposing these conditions results in the following four relationships:

$$A_1' = B_1' + C_1'b^3 + E_0'b^3$$
$$A_1'' = B_1'' + C_1''b^3 + E_0''b^3$$
$$F_1' = C_1' + B_1'a^{-3}$$
$$F_1'' = C_1'' + B_1''a^{-3}$$

At each interface, the normal component of the total current density must be continuous. $J_r = gE_r + \varepsilon\partial E_r/\partial t$ where $E_r = -\partial V/\partial r$ and g_i and ε_i are the conductivity and permittivity of medium i.

This requirement yields four additional relationships:

$$g_1A_1' - g_2B_1' + \omega\varepsilon_1A_1'' - \omega\varepsilon_2B_1'' = -b^3/2(\omega\varepsilon_2C_1'' + g_2C_1' + g_1E_0')$$
$$g_1A_1'' - g_2B_1'' - \omega\varepsilon_1A_1' + \omega\varepsilon_2B_1' = +b^3/2(\omega\varepsilon_2C_1' - g_2C_1'' + \omega\varepsilon_1E_0')$$
$$g_2B_1' + \omega\varepsilon_2B_1'' = a^3/2(g_2C_1' + \omega\varepsilon_2C_1'' - g_3F_1' - \omega\varepsilon_3F_1'')$$
$$g_2B_1'' - \omega\varepsilon_2B_1' = a^3/2(g_2C_1'' - \omega\varepsilon_2C_1' - g_3F_1'' + \omega\varepsilon_3F_1')$$

Solving the eight equations in eight unknowns yields the following results:

$$A_1' = (G_2 + b^3)C_1' + G_1C_1'' + E_0'b^3$$
$$A_1'' = (G_2 + b^3)C_1'' - G_1C_1' + E_0''b_3$$
$$B_1' = G_2C_1' + G_1C_1''$$
$$B_1'' = G_2C_1'' - G_1C_1'$$
$$C_1' = -\frac{S_1R_1 + S_2R_2}{R_1^2 + R_2^2}$$
$$C_1'' = \frac{S_2R_1 - S_1R_2}{R_1^2 + R_2^2}$$
$$F_1' = (1 + G_2/a^3)C_1' + G_1C_1''/a^3$$
$$F_1'' = (1 + G_2/a^3)C_1'' - G_1C_1'/a^3$$

where

$$Z_{23} = a^3/[\omega^2(2\varepsilon_2 + \varepsilon_3)^2 + (2g_2 + g_3)^2]$$
$$G_1 = 3\omega Z_{23}(\varepsilon_2g_3 - \varepsilon_3g_2)$$
$$G_2 = Z_{23}[\omega^2(\varepsilon_2 - \varepsilon_3)(2\varepsilon_2 + \varepsilon_3) + (g_2 - g_3)(2g_2 + g_3)]$$
$$R_1 = G_1(g_2 - g_1) + \omega G_2(\varepsilon_2 - \varepsilon_1) - \omega b^3(\varepsilon_1 + \varepsilon_2/2)$$
$$R_2 = G_2(g_2 - g_1) - \omega G_1(\varepsilon_2 - \varepsilon_1) - b^3(g_1 + g_2/2)$$
$$S_1 = b^3(\omega\varepsilon_1E_0'' - 3g_1E_0'/2)$$
$$S_2 = b^3(3\omega\varepsilon_1E_0'/2 - g_1E_0'')$$

The expressions for the actual fields in the sphere and shell are:

$$\boldsymbol{E}_3 = -F_1'\cos\omega t\boldsymbol{l}_z - F_1''\sin\omega t\boldsymbol{l}_z$$
$$\boldsymbol{E}_2 = [\boldsymbol{l}_r\cos\theta + \boldsymbol{l}_\theta\sin\theta](B_1'\cos\omega t + B_1''\sin\omega t)/r^3 - (C_1'\cos\omega t + C_1''\sin\omega t)\boldsymbol{l}_z$$

3 Application of model

3.1 *General behaviour*

The relationship between tissue characteristics and the resulting fields can be seen by considering an object exposed to an extremely low frequency (ELF) field in air. In this case, $g_1 = 0$, and only the changes in the conductivities of the two layers are significant. Figs. 2 and 3 illustrate the peak fields in the inner core (left side) and outer shell for two widely different values of a/b. The inner-core field is uniform, and the outer shell field may vary with position for $a \ll b$ (Fig. 2). For the even numbered curves the conductivity of the core is much greater than that of the shell and the field in the shell increases close to the core. A similar situation occurs when a conducting sphere is placed in air. In this case, the field lines converge near the sphere, and the field in the air near the sphere

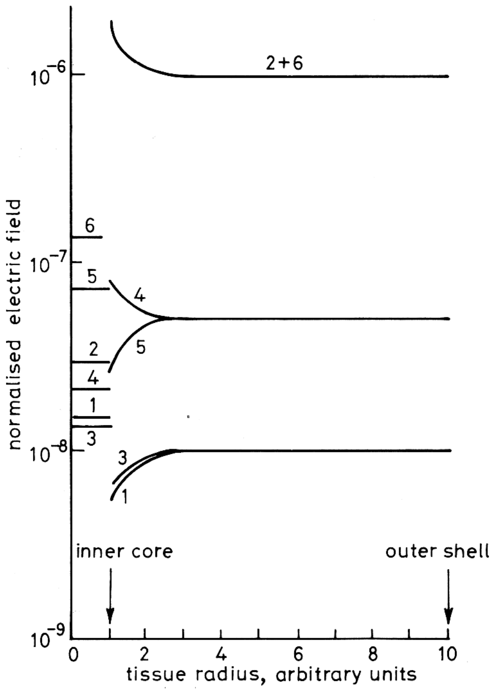

Fig. 2 *Normalised electric field (the electric field in the tissue divided by the applied electric field) for tissue exposed in air: a = 1 and b = 10*

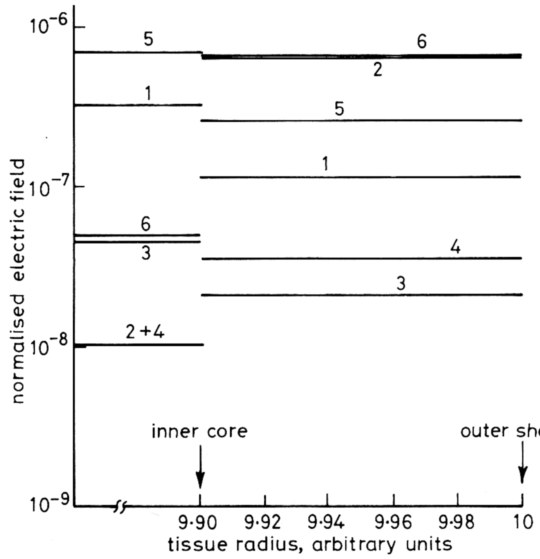

Fig. 3 *Normalised electric field for tissue exposed in air: a = 9.9 and b = 10*

increases. For the odd numbered curves, the conductivity of the core is much less than that of the shell. This corresponds to an insulating sphere embedded in a conductor. The field lines diverge near the insulator, and the field in the conductor is therefore weaker in the vicinity of the insulator.

By comparing Figs. 2 and 3, it can be seen that a thicker outer shell provides electrical shielding for the core. This phenomenon is shown more clearly in Fig. 4 in which the inner-shell field is plotted against the fractional outer-shell thickness $f = 1 - a/b$. As $a \rightarrow 0$ (the outer shell gets thicker) the field in the core decreases for the cases in which the conductivity of the shell exceeds that of the core. Thus a conducting shell will reduce but not eliminate the field in a poorly conducting core. The opposite dependence of the field on f occurs when the core is surrounded by an insulating layer (curves 2, 4, and 6).

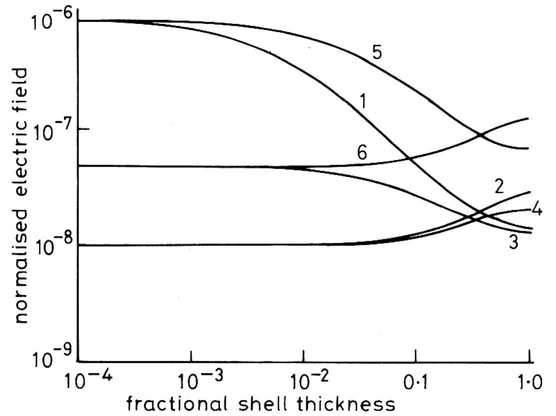

Fig. 4 *Normalised electric field in inner core as a function of fractional shell thickness*

3.2 *Specific applications*

We have applied the model to a related series of studies involving physiological changes in the central nervous system (HANSSON, 1981; HAMER, 1968; BAWIN and ADEY, 1976), and to a report involving effects on amoebas (FRIEND and FINCH, 1975). Hansson found histopathological changes in rabbit brain following chronic exposure to 60 Hz electric fields at 14 000 Vm^{-1}. Assuming the values for relative permittivity and conductivity shown in Table 1 (ROBILLARD and POUSSART, 1977; LAKES *et al.,* 1977) the electric field is found to be $1{\cdot}8 \times 10^{-2}$ Vm^{-1} and $1{\cdot}3 \times 10^{-3}$ Vm^{-1} in the bone and brain tissue, respectively. The field in the brain tissue was almost five orders of magnitude greater than that present in human brain upon brief application of an electric field of 4 Vm^{-1}, 7 Hz which altered human reaction time (HAMER, 1968) (see Table 2). *In vitro* studies involving exposure of isolated brain tissue exposed for 20 min to 56 Vm^{-1}, 32 Hz apparently have resulted in altered calcium exchange between

Table 1 Sets of assumed tissue constants used to illustrate general behaviour of the solution. The set numbers correspond to the similarly labelled curves in Figs. 2-4

	Outer shell		Inner core	
Set number	Relative permittivity	Conductivity, S m^{-1}	Relative permittivity	Conductivity, S m^{-1}
1	80	1	7×10^5	0·01
2	7×10^5	0·01	80	1
3	80	1	8×10^6	0·2
4	8×10^6	0·2	80	1
5	8×10^6	0·2	7×10^5	0·01
6	7×10^5	0·01	8×10^6	0·2

Table 2 Calculated tissue electric fields associated with various reported biological effects

Experimental system	Rabbit brain	Human brain	Brain tissue	Amoeba
Applied electric field, V m^{-1}	14 000 (in air)	4 (in air)	56 (in air)	2500 (in solution)
Frequency	60 Hz	7Hz	32	5 MHz
Reference	HANSSON, 1981	HAMER, 1968	BAWIN and ADEY, 1976	FRIEND and FINCH, 1975
Application of model	brain tissue surrounded by bone	brain tissue surrounded by bone	brain tissue	cell cytoplasm surrounded by cell membrane
Outer medium	air	air	aqueous solution	aqueous solution
Core radius, mm	5	75	2·5	0·5
Shell thickness, mm	3	4		6×10^{-6}
Relative permittivity	1 (air) 10^4–10^6 (bone) 10^5–10^7 (brain)	1 (air) 10^4–10^6 (bone) 10^5–10^7 (brain)	81 (external medium) 10^5–10^7 (brain)	81 (external medium) 2·5–25 (cell membrane) 81 (cytoplasm)
Conductivity, S m^{-1}	0 (air) 10^{-2} (bone) 0·2 (brain)	0 (air) 10^{-2} (bone) 0·1 (brain)	1 (external medium) 0·2 (brain)	1 (external medium) 10^6–10^{10} (cell membrane) 1 (cytoplasm)
Electric field, Vm^{-1}	$1{\cdot}8 \times 10^{-2}$ (bone) $1{\cdot}3 \times 10^{-3}$ (brain)	$3{\cdot}5 \times 10^{-7}$ (bone) $5{\cdot}1 \times 10^{-8}$ (brain)	$4{\cdot}3 \times 10^{-7}$ (brain)	$0{\cdot}5 \times 10^6$ (cell membrane) $2{\cdot}5 \times 10^3$ (cytoplasm)

the brain tissue and its aqueous environment (BAWIN and ADEY, 1976). The calculated value of the electric field in the brain tissue in these experiments is given in Table 1. Thus chronic exposure to an electric field that produced a brain-tissue field of $1{\cdot}3 \times 10^{-3}$ V m^{-1} resulted in morphological changes, but brief exposures that produced a brain-tissue field of $5{\cdot}1 \times 10^{-8}$ V m^{-1} yielded only a less severe functional change.

Application of electric fields to amoebas produced a variety of morphological changes following relatively brief exposures (FRIEND and FINCH, 1975). Application of the model clearly demonstrates that large electric fields were present in the cytoplasm as well as the cell membrane.

References

BASSETT, C. A. L., MITCHELL, S. N. and GASTON, S. R. (1982) Pulsing electromagnetic field treatment in ununited fractures and failed arthrodeses. *JAMA,* **247,** 623-628.

BAWIN, S. M. and ADEY, W. R. (1976) Sensitivity of calcium binding in cerebral tissue to weak environmental electric fields oscillating at low frequency. *Proc. Nat. Acad. Sci. USA,* **73,** 1999-2003.

BECKER, R. 0. and MARINO, A. A. (1982) *Electromagnetism and life*. State University of New York Press, Albany, New York.

FRIEND, A. W. Jr. and FINCH, E. D. (1975) Low frequency electric field induced changes in the shape and motility of amoebas. *Science,* **187,** 357-358.

HAMER, J. R. (1968) Effects of low level, low frequency electric fields on human reaction time. *Comm. Behav. Biol.,* Part A, **2,** 217-222.

HANSSON, H. (1981) Lamellar bodies in Purkinje nerve cells experimentally induced by electric field. *Brain Res.,* **216,** 187–191.

HART, F. X. and MARINO, A. A. (1976) Biophysics of animal response to an electrostatic field. *J. Biol. Phys.,* **4,** 124-142.

LAKES, R. S., HARPER, R. A. and KATZ, J. L. (1977) Dielectric relaxation in cortical bone. *J. Appi. Phys.,* **48,** 808-811.

MARCHESI, M. and PARODI, M. (1982) Study of the electrical field inside biological structures. *Med. & Biol. Eng. & Comput.,* **20,** 608-612.

MARINO, A. A., BERGER, T. J., AUSTIN, B. P., BECKER, R. O. and HART, F. X. (1977) *In vivo* bioelectrochemical changes associated with exposure to extremely low frequency electric fields. *Phys. Chem. Phys.,* **9,** 433-441.

MARINO, A. A., CULLEN, J. M., REICHMANIS, M., BECKER, R. O. and HART, F. X. (1980) Sensitivity to change in electrical environment: a new bioelectric effect. *Am. J. Physiol.,* **239,** 424–427.

ROBILLARD, P. N. and POUSSART, Y. (1977) Specific-impedance measurements of brain tissues. *Med. & Biol. Eng. & Comput.,* **15,** 438-445.