Energy Flux Along High Voltage Transmission Lines

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Abstract—The ground level energy flux profile for an EHV overhead transmission line has been calculated. The results provide a reference point for the choice of appropriate exposure levels is biological experimentation involving health and safety aspects of EHV transmission lines. Large energy fluxes were found at points beyond the edge of the ROW.

INTRODUCTION

There are thousands of miles of overhead transmission lines in service at the present time in the United States in the 345-765 kV range, and more are planned. Research is in progress to study the environmental compatibility of such lines, particularly their electric and magnetic fields. To facilitate such research, we present herein a calculation of the energy flux profile of a modern three-phase EHV overhead transmission line. It is hoped that this information will aid investigators in the choice of appropriate exposure levels for biological experimentation.

RESULTS

The ground level energy flux profile of the conductor distribution shown in Figure 1 will be calculated. It will be assumed throughout, that the earth is a good conductor.

Electric Field

The electric field of the conductor distribution shown in Figure 1 may be found by the method of images. Consider the first array of such conductors as shown in Figure 2. The potential at some point, P, can be written as the sum of the potentials due to the individual wires:

$$\phi(P) = \frac{1}{2\Pi\epsilon_0}(q_1 \ln(r_1'/r_1) + q_2 \ln(r_2'/r_2) + q_3 \ln(r_3'/r_3) + q_4 \ln(r_4'/r_4)) \quad (1)$$

where q_i is the linear charge density on the i^{th} wire, and ϵ_0 is the permittivity of free space. By setting $\phi = V_1$ at the location of each wire in turn, and solving the resulting set of four simultaneous equations, it can be shown that the q_i are equal, for $d \gg a$. These results can be generalized to the case of three arrays of four conductors shown in Figure 1. If d, $c \gg a$, each array can be replaced by a single wire having a charge density four times that of the individual wires within each array. Let $Q_i = 4q_i$ be the total linear charge density of the i^{th} array. The potential at the surface of each array can be expressed as the sum of contributions, given by equation 1, from the three arrays:

$$\begin{aligned} V_1 &= Q_1C_1 + Q_2C_2 + Q_3C_3 \\ V_2 &= Q_1C_2 + Q_2C_1 + Q_3C_2 \\ V_3 &= Q_1C_3 + Q_2C_2 + Q_3C_1 \end{aligned} \quad (2)$$

where:

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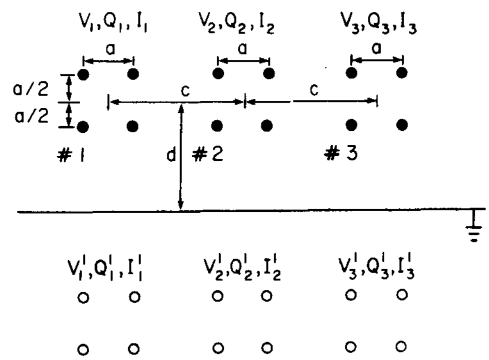

Figure 1. Proposed 765 kV overhead transmission line (*I*) showing various quantitites used in computation. V_i, Q_i, I_i, are voltage charge density, and current respectively for the *i*th array. Primed quantities are corresponding values for the image arrays.

$$C_1 = \frac{1}{2\pi\epsilon_0} \ln\left(16d^4/a^3 b(2)^{1/2}\right)$$

$$C_2 = \frac{4}{2\pi\epsilon_0} \ln\left(c^{-1}(4d^2 + c^2)^{1/2}\right)$$

$$C_3 = \frac{4}{2\pi\epsilon_0} \ln\left(c^{-1}(d^2 + c^2)^{1/2}\right)$$

where b is the radius of an individual wire in each array. Equations (2) can be solved simultaneously to yield:

$$Q_1 = (C_6 V_1 - C_5 V_2 - K_3 V_3)/K_4$$

$$Q_2 = (V_1 - Q_1 C_7 - C_3 V_3/C_1)/C_5$$

$$Q_3 = (V_3 - Q_1 C_3 - Q_2 C_2)/C_1$$

where:

$$C_5 = C_2 - C_3 C_2/C_1$$

$$C_6 = C_1 - C_2^2/C_1$$

$$C_7 = C_1 - C_3^2/C_1$$

$$K_3 = (C_6 C_3 - C_5 C_2)/C_1$$

$$K_4 = C_6 C_7 - C_5^2.$$

The electric field at a ground-level point, P, in Figure 3 is the sum of the three individual fields, subject to the boundary condition that the horizontal component of E_{total} vanishes at the earth's surface. These are:

$$E_1 = \frac{Q_1}{2\pi\epsilon_0 r_1}, \quad r_1 = ((x - c)^2 + d^2)^{1/2}$$

$$E_2 = \frac{Q_2}{2\pi\epsilon_0 r_2}, \quad r_2 = (x^2 + d^2)^{1/2}$$

$$E_3 = \frac{Q_3}{2\pi\epsilon_0 r_3}, \quad r_3 = ((x + c)^2 + d^2)^{1/2}$$

with similar expressions for the image fields E_1', E_2', and E_3'. Therefore,

$$E_{\text{total}} = 2(E_1 \sin\theta_1 + E_2 \sin\theta_2 + E_3 \sin\theta_3),$$

or

$$E_{\text{total}} = \frac{d}{\pi\epsilon_0} (Q_1/r_1^2 + Q_2/r_2^2 + Q_3/r_3^2). \tag{3}$$

We now introduce the phase relationship among the arrays.

$$V_1 = V \cos\omega t$$

$$V_2 = V \cos(\omega t + 120°)$$

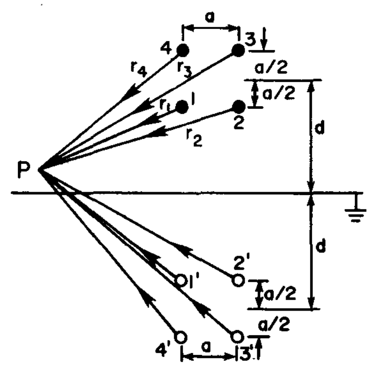

Figure 2. Array of four conductors and image array.

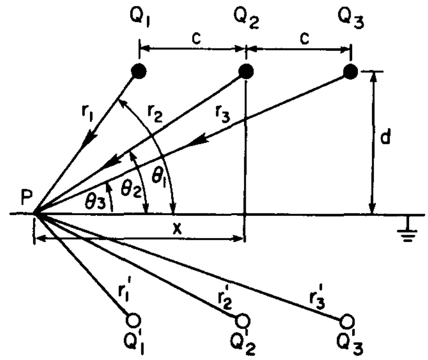

Figure 3. Equivalent transmission line geometry.

$$V_3 = V \cos(\omega t - 120°)$$

where V is the peak voltage relative to ground. It can be shown that equation (3) becomes:

$$E(x) = \frac{4dV}{\pi\epsilon_0} (A_6 \cos\omega t + B_6 \sin\omega t) \tag{4}$$

where:

$$A_6 = A_1/r_1^2 + A_2/r_2^2 + A_3/r_3^2$$

$$B_6 = B_1/r_1^2 + B_2/r_2^2 + B_3/r_3^2$$

$$A_1 = (C_6 + 0.5C_5 + 0.5K_3)/K_4$$

$$B_1 = 0.866(C_5 - K_3)/K_4$$

$$A_2 = (1 - C_7 A_1 + 0.5C_3/C_1)/C_5$$

$$B_2 = (-C_7 B_1 - 0.866C_3/C_1)/C_5$$

$$A_3 = (-0.5 - C_3 A_1 - C_2 A_2)/C_1$$

$$B_3 = (0.866 - C_3 B_1 - C_2 B_2)/C_1.$$

Magnetic Field

The method of images may also be used to calculate the magnetic field. The boundary conditions require that the perpendicular component of the field vanish at the earth's surface, thus only the tangential component need be computed to find the ground level field. The total ground level field is simply twice the horizontal component from each group of conductors, which for simplicity will be considered to be single conductors rather than square arrays. From Figures 1 and 3 it can be seen that:

$$H = \frac{d}{\pi} (I_1/r_1^2 + I_2/r_2^2 + I_3/r_3^2). \tag{5}$$

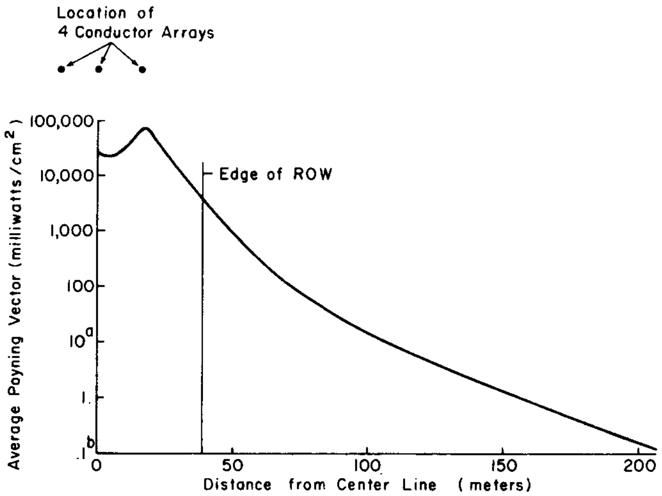

Figure 4. Average ground level Poynting Vector as a function of lateral distance from the center line for the proposed Pannell Road-Sterling-Volney 765 kV Transmission Line (1). $I_{rms} = 4000$ A, $V_{rms} = 443$ kV, $a = 0.45$ m, $b = 1.75 \times 10^{-2}$ m, $c = d = 15$ m: [a]U.S. occupational microwave exposure standard: [b]Soviet microwave exposure standard (2).

If the resistances and shunt conductances are negligible, the current in each conductor will be in phase with the corresponding voltage. If I is the peak current, we have:

$$I_1 = I \cos \omega t$$
$$I_2 = I \cos (\omega t + 120^\circ)$$
$$I_3 = I \cos (\omega t - 120^\circ). \qquad (6)$$

Employing equations (6), equation (5) may be written:

$$H(x) = \frac{dI}{\pi} (A_7 \cos \omega t + B_7 \sin \omega t) \qquad (7)$$

where:

$$A_7 = 1/r_1^2 - 0.5/r_2^2 - 0.5/r_3^2$$
$$B_7 = -0.866/r_2^2 + 0.866/r_3^2.$$

Energy Flux

The ground level energy flux, or Poynting Vector, $\overline{S} = \overline{E} \times \overline{H} = E(x)H(x)$, can be computed from equations (4) and (7).

$$S(x,t) = \frac{4d^2 IV}{\pi^2 \epsilon_0} (A_6 A_7 \cos^2 \omega t + B_6 B_7 \sin^2 \omega t + (B_6 A_7 + B_7 A_6) \sin \omega t \cos \omega t).$$

The time averaged Poynting Vector, defined as:

$$S(x) = \int_0^T S(x,t)\, dt \Big/ \int_0^T dt, \qquad T = 2\pi/\omega$$

can be shown to be:

$$S(x) = \frac{4d^2 I_{rms} V_{rms}}{\pi^2 \epsilon_0} (A_6 A_7 + B_6 B_7). \qquad (8)$$

Equation (8) is displayed graphically in Figure 4.

Discussion

The three-phase system tends to average out time variations of the Poynting Vector; consequently, lateral profiles of the maximum and average value of the energy flux are identical to within about 3%.

The energy transmitted by overhead transmission lines exists in the space surrounding the wires. It is perhaps surprising to note the large energy flux which exists beyond the ROW. Considerable attention has been paid to the hazards of microwave irradiation, whereas the consequences of exposure to ELF fields have only recently come under study. Unlike the situation for microwaves, there are currently no human exposure standards for the ELF region. From Figure 4, the ELF energy flux does not decrease to the level of the United States microwave exposure standard until about 108 m from the center line of the ROW. The corresponding distance for the Soviet microwave exposure standard is about 210 m. While the United States standard is based exclusively on simple physiological considerations involving tissue heating (2), the Soviet protection level clearly envisions other possible mechanisms. The extent to which such mechanisms are also operative at 60 Hz, and the existence of mechanisms peculiar to the ELF region, are both presently undetermined.

References

(1) Application to the State of New York Public Service Commission for Certificate of Environmental Compatibility and Public Need, Rochester Gas And Electric Corporation, Niagara Mohawk Power Corporation, January 1974.

(2) S. M. Michaelson, "Microwave Exposure Safety Standards–Physiologic and Philosophic Aspects," *American Industrial Hygiene Association Journal*, vol. 33, pp. 156-164, March 1972.