JOURNAL OF BIOELECTRICITY, 8(2), 147-158 (1989)

EXPOSURE SYSTEM FOR PRODUCTION OF UNIFORM MAGNETIC FIELDS

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ABSTRACT

An expression was derived for the magnetic field between Helmholtz coils, and used to determine the intercoil region within which the field's spatial uniformity is below an arbitrary value. Coils were designed to produce a magnetic field uniform to within 5% in a specified volume, and the results of the calculations were verified by direct measurement.

INTRODUCTION

The magnetic field produced by a current-carrying coil is spatially nonuniform. When two planar circular coils are arranged coaxially and parallel to one another, they can be connected electrically so that their magnetic fields are additive in the region between the coils. If the separation between the coils is equal to the coil radius, the magnitude of the magnetic field is independent of position along their common axis (except at points near the coils) (1). Coils satisfying this condition, termed Helmholtz coils, are frequently used for exposing biological systems such as cells (2,3) and human subjects (4,5) under the assumption that the field in the region occupied by the biological system is relatively uniform.

The use of paired coils to generate uniform magnetic fields raises the problem of the coil radius required to insure that the percent variation of magnetic field within a given volume between the coils

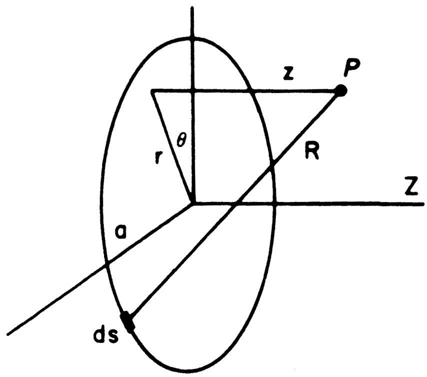

FIGURE 1. Magnetic field generated by a current loop.

is less than a preassigned value. The solution to this problem, under the arbitrary condition that the coils are Helmholtz coils, is presented here.

EQUATIONS FOR MAGNETIC FIELD

Consider the current loop shown in Figure 1. From the law of Biot and Savart:

$$\mathbf{B} = del \times \mathbf{A} = del \times \frac{\mu_o I}{4\pi} \int \frac{\mathbf{ds}}{R}$$

where **A** is the vector potential, **B** is the magnetic induction (Teslas), del is the gradient, μ_o is the permeability of free space ($4\pi x 10^{-7}$ H/m), I is the current through the loop (amperes), R is the distance from a point of integration on the circuit to P, the point of observation (meters), and **ds** is an infinitesimal segment of the current loop.

From symmetry, **A** is independent of θ and has only the single component A_θ. From Figure 1

$$A_\theta = \frac{\mu_o I a}{4\pi} \int_0^{\pi} \frac{\cos\theta d\theta}{\left(z^2 + r^2 + a^2 - 2ra\cos\theta\right)^{1/2}}.$$

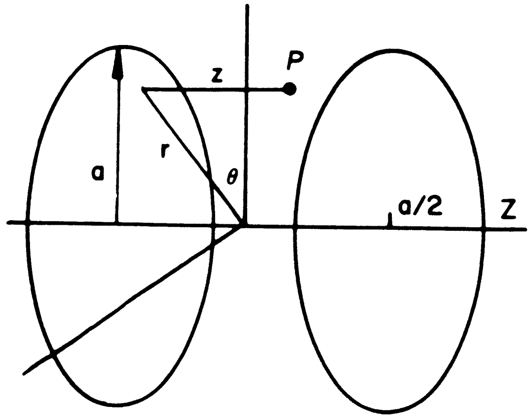

FIGURE 2. Position of Helmholtz coils.

The integral cannot be evaluated in terms of elementary functions, but it can be written in a form involving elliptical integrals.

$$A_0 = \frac{\mu_o I a^{1/2}}{2\pi r^{1/2}}\left[(2/k - k)K(k) - (2/k)E(K)\right]$$

where

$$K(k) = \int_0^{\pi/2}\left(1 - k^2 \sin^2\alpha\right)^{-1/2} d\alpha$$

$$E(k) = \int_0^{\pi/2}\left(1 - k^2 \sin^2\alpha\right)^{1/2} d\alpha$$

$$k \equiv \left[4ra/\left(z^2 + (r+a)^2\right)\right]^{1/2}$$

Since **B** = del x **A**,

$$B_r = \frac{\mu_0 I}{2\pi}\frac{z}{r\left[(a+r)^2 + z^2\right]^{1/2}}\left[-K(k) + \frac{a^2 + r^2 + z^2}{(a-r)^2 + z^2}E(k)\right]$$

$$B_z = \frac{1}{\left[(a+r)^2 + z\right]^{1/2}}\left[K(k) + \frac{a^2 - r^2 - z^2}{(a-r)^2 + z^2}E(k)\right]$$

$$\mathbf{B}_{\mathrm{Total}} = \mathbf{B}_r + \mathbf{B}_z$$

For any given value of a, picking a point, P, fixes r and z, and hence determines k; K(k) and E(k) can then be found from tables (6).

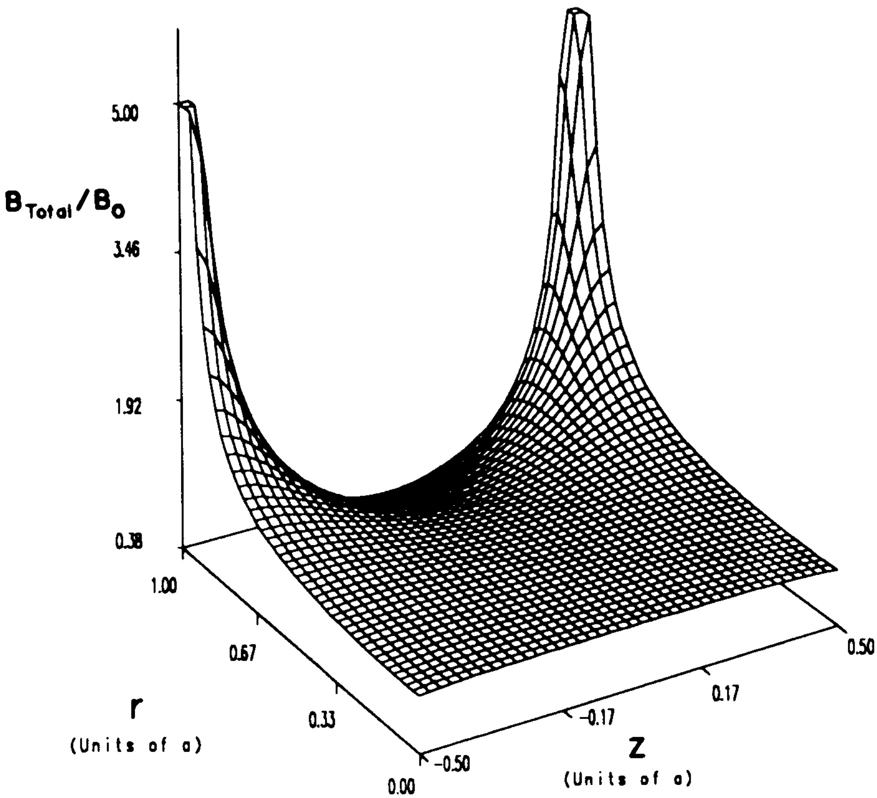

FIGURE 3. Total magnetic field (normalized by B_0) as a function of position between Helmholtz coils located at z = ± 0.5a.

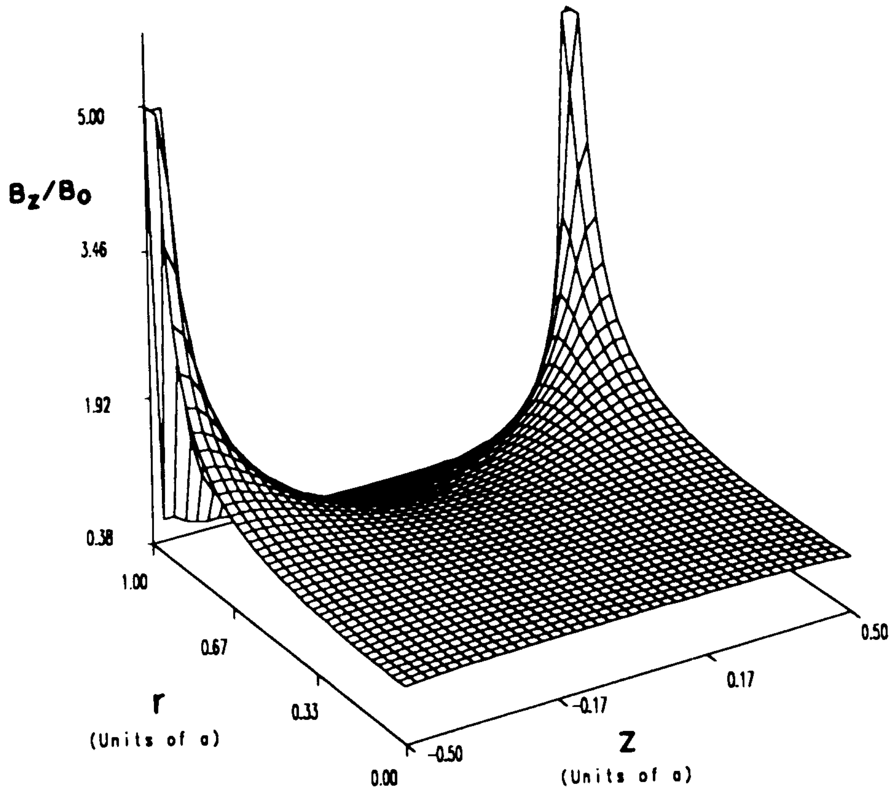

FIGURE 4. Axial component of the magnetic field (normalized by B_0) as a function of position between Helmholtz coils located at z = ± 0.5a.

The equations for B_r and B_z can be used to calculate the magnetic field between Helmholtz coils (Figure 2).

Right coil:

$$B_{r1} = \frac{\mu_0 I}{2\pi} \frac{z - a/2}{r\left[(a+r)^2 + (z-a/2)^2\right]^{1/2}} \left[-K(k) + \frac{a^2 + r^2 + (z-a/2)^2}{(a-r)^2 + (z-a/2)^2} E(k)\right]$$

$$B_{z1} = \frac{\mu_0 I}{2\pi} \frac{1}{\left[(a+r)^2 + (z-a/2)^2\right]^{1/2}} \left[-K(k) + \frac{a^2 + r^2 - (z-a/2)^2}{(a-r)^2 + (z-a/2)^2} E(k)\right]$$

Left coil:

$$B_{r2} = \frac{\mu_0 I}{2\pi} \frac{z + a/2}{r\left[(a+r)^2 + (z+a/2)^2\right]^{1/2}} \left[-K(k) + \frac{a^2 + r^2 + (z+a/2)^2}{(a-r)^2 + (z+a/2)^2} E(k)\right]$$

$$B_{z2} = \frac{\mu_0 I}{2\pi} \frac{1}{\left[(a+r)^2 + (z+a/2)^2\right]^{1/2}} \left[-K(k) + \frac{a^2 - r^2 - (z+a/2)^2}{(a-r)^2 + (z+a/2)^2} E(k)\right]$$

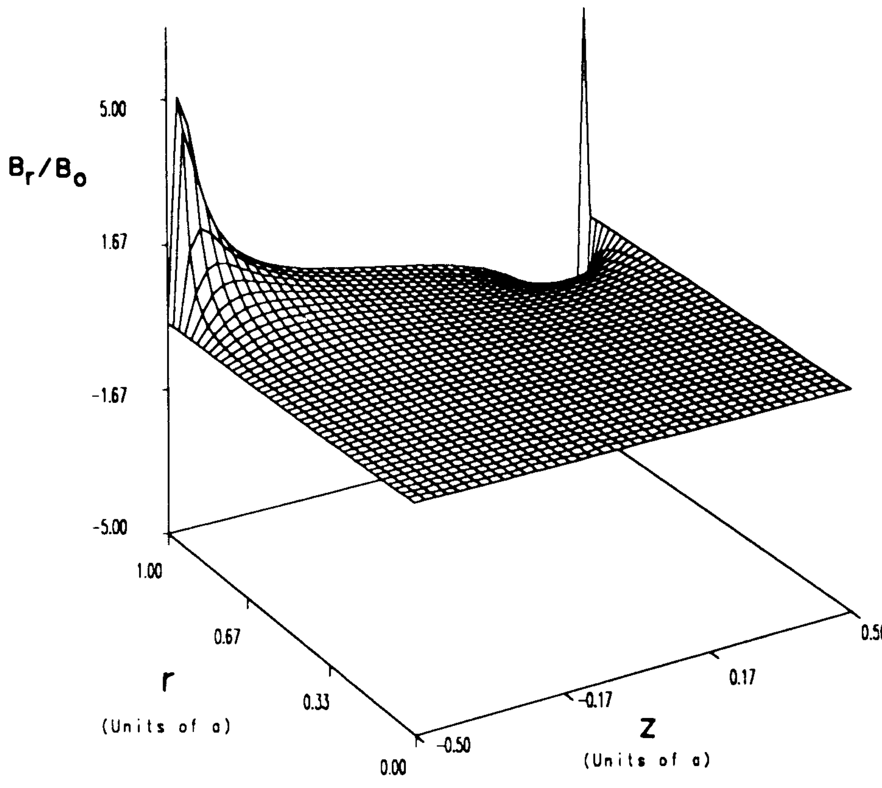

FIGURE 5. Radial component of the magnetic field (normalized by B_0) as a function of position between Helmholtz coils located at $z = \pm 0.5a$.

TABLE 1. Magnetic Field at Various Points in the Intercoil Region. r and z are measured in units of the coil radius, a. B_r and B_z are the magnetic field in the radial and axial directions, respectively, in units of B_o. $B_{Total} = (B_z^2 + B_r^2)^{1/2}$.

z	r	B_z	B_r	B_{Total}
0.0	0.0	1.000	1.000	0.000
0.0	0.1	1.006	1.000	0.000
0.0	0.2	0.998	0.998	0.000
0.0	0.3	0.997	0.997	0.000
0.0	0.4	0.986	0.986	0.000
0.0	0.5	0.964	0.964	0.000
0.0	0.6	0.929	0.929	0.000
0.0	0.7	0.853	0.853	0.000
0.1	0.0	1.000	1.000	0.000
0.1	0.1	1.000	1.000	0.009
0.1	0.2	1.001	1.001	0.003
0.1	0.3	1.000	1.000	0.002
0.1	0.4	0.993	0.993	0.013
0.1	0.5	0.981	0.980	0.023
0.1	0.6	0.948	0.946	0.052
0.1	0.7	0.883	0.878	0.094
0.2	0.0	0.998	0.998	0.000
0.2	0.1	1.000	1.000	0.005
0.2	0.2	1.002	1.002	0.000
0.2	0.3	1.008	1.008	0.006
0.2	0.4	1.013	1.013	0.016
0.2	0.5	1.017	1.016	0.040
0.2	0.6	1.011	1.007	0.092
0.2	0.7	0.989	0.975	0.168
0.3	0.0	0.992	0.992	0.000
0.3	0.1	0.994	0.994	0.004
0.3	0.2	1.001	1.000	0.011
0.3	0.3	1.014	1.014	0.004
0.3	0.4	1.032	1.032	0.008
0.3	0.5	1.057	1.057	0.025
0.3	0.6	1.093	1.090	0.080
0.3	0.7	1.138	1.123	0.186
0.4	0.0	0.975	0.975	0.000
0.4	0.1	0.978	0.978	0.019
0.4	0.2	0.990	0.990	0.015
0.4	0.3	1.009	1.009	0.025
0.4	0.4	1.039	1.038	0.032
0.4	0.5	1.088	1.088	0.017
0.4	0.6	1.166	1.166	0.019
0.4	0.7	1.290	1.286	0.096

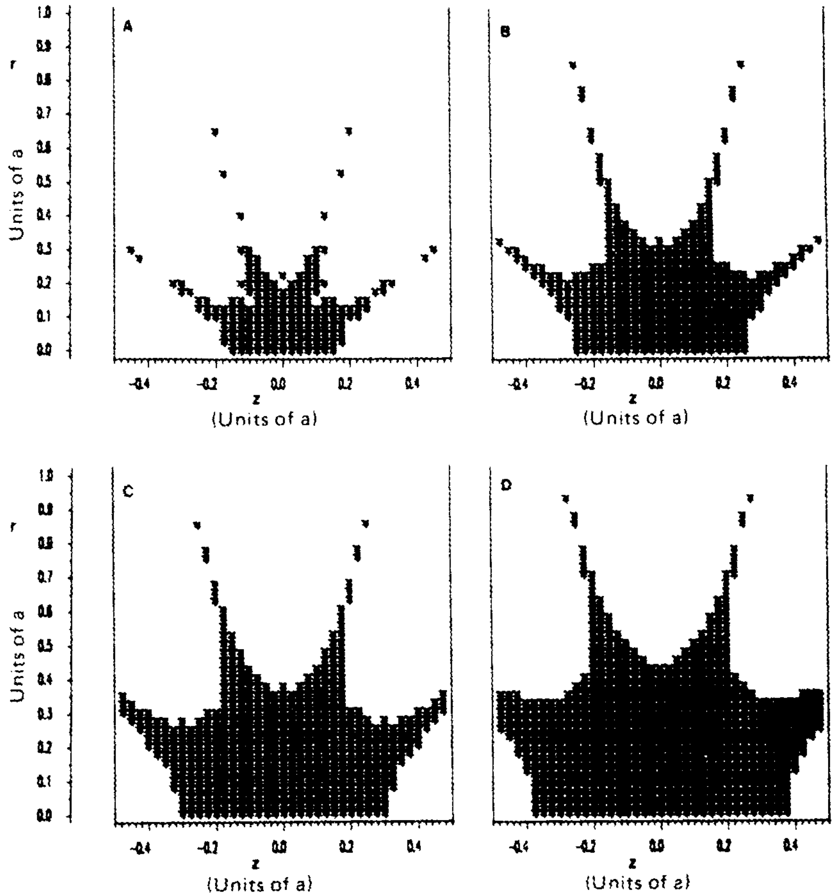

FIGURE 6. A-D, respectively: region in which B_{Total} is uniform within 0.1%, 0.5%, 1%, 2%. The solid lines depict the coil locations.

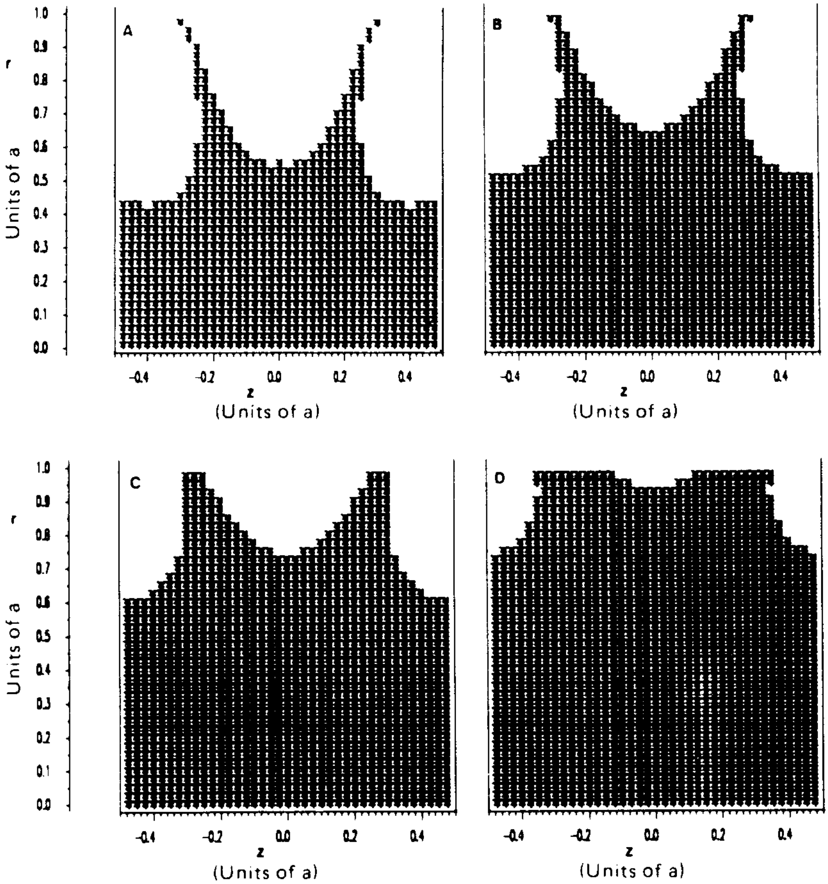

FIGURE 7. A-D, respectively: region in which B_{Total} is uniform within 5%, 10%, 20%, 50%. The solid lines depict the coil locations.

TABLE 2. Experimental Verification of Magnetic Field Produced by Helmholtz Coils Having A Radius of 0.65 m. Measurements were made for B_{Total} = 0.1 mT, 60 Hz.

LOCATION		MEASURED		THEORETICAL		% ERROR	
z	r	B_z	B_{Total}	B_z	B_{Total}	B_z	B_{Total}
0.0	0.0	1.011	1.011	1	1	1.1	1.1
0.0	0.1	1.012	0.013	1.000	1.000	1.14	1.24
0.0	0.2	1.011	0.012	0.998	0.998	1.29	1.39
0.0	0.3	1.008	0.009	0.997	0.997	1.13	1.23
0.0	0.4	0.999	0.001	0.986	0.986	1.33	1.54
0.0	0.5	0.977	0.979	0.964	0.964	1.36	1.56
0.0	0.6	0.936	0.938	0.929	0.929	0.76	0.97
0.0	0.7	0.858	0.858	0.853	0.853	0.54	0.54
0.1	0.0	1.012	1.012	1.000	1.000	1.21	1.21
0.1	0.1	1.014	1.014	1.000	1.000	1.39	1.38
0.1	0.2	1.015	1.015	1.001	1.001	1.41	1.41
0.1	0.3	1.016	1.017	1.000	1.000	1.56	1.66
0.1	0.4	1.012	1.014	0.993	0.993	1.87	2.07
0.1	0.5	0.998	1.000	0.980	0.981	1.80	1.98
0.1	0.6	0.964	0.968	0.946	0.948	1.89	2.16
0.1	0.7	0.887	0.896	0.878	0.883	1.04	1.49
0.2	0.0	1.011	1.011	0.998	0.998	1.28	1.28
0.2	0.1	1.014	1.014	1.000	1.000	1.44	1.44
0.2	0.2	1.019	1.019	1.002	1.002	1.64	1.64
0.2	0.3	1.027	1.027	1.008	1.008	1.91	1.91
0.2	0.4	1.035	1.037	1.013	1.013	2.20	2.38
0.2	0.5	1.040	1.043	1.016	1.017	2.32	2.54
0.2	0.6	1.031	1.040	1.007	1.011	2.41	2.88
0.2	0.7	0.990	1.012	0.975	0.989	1.57	2.32
0.3	0.0	1.005	1.005	0.992	0.992	1.36	1.36
0.3	0.1	1.008	1.008	0.994	0.994	1.40	1.40
0.3	0.2	1.019	1.020	1.000	1.001	1.84	1.93
0.3	0.3	1.037	1.037	1.014	1.014	2.30	2.30
0.3	0.4	1.060	1.060	1.032	1.032	2.66	2.66
0.3	0.5	1.095	1.096	1.057	1.057	3.62	3.69
0.3	0.6	1.136	1.144	1.090	1.093	4.18	4.63
0.3	0.7	1.172	1.191	1.123	1.138	4.38	4.64
0.4	0.0	0.985	0.985	0.975	0.975	0.99	0.99
0.4	0.1	0.992	0.992	0.978	0.978	1.39	1.37
0.4	0.2	1.006	1.006	0.990	0.990	1.60	1.59
0.4	0.3	1.034	1.034	1.009	1.009	2.47	2.44
0.4	0.4	1.071	1.071	1.038	1.039	3.13	3.08
0.4	0.5	1.132	1.133	1.088	1.088	4.01	4.09
0.4	0.6	1.215	1.216	1.166	1.166	4.17	4.24
0.4	0.7	1.341	1.346	1.286	1.290	4.26	4.36

REGIONS OF UNIFORMITY

The field uniformity between Helmholtz coils can be conveniently depicted using B_0 = B_{Total}(z=0, r=0) as a reference. Figure 3 shows B_{Total}10/B_0 as a function of r and z. Comparable values for B_z/B_0 and B_r/B_0 are given in Figures 4 and 5, respectively. B_r makes little contribution to the magnetic field except at the edge of the intercoil region (Table 1). Figures 6A-D show values of r and z for which B_{Total} is within 0.1%, 0.5%, 1% and 2% of B_0, respectively. Figures 7A-D show comparable values for B_{Total} within 5%, 10%, 20% and 5% of B_0, respectively.

COIL DESIGN AND TESTING

Coils were designed to produce a magnetic field uniform within 5% in a cylindrical volume of radius 0.27 m. From Figure 7A, this condition is met when r $\leq$ 0.42a. For r = 0.27 m, we have a = 0.65 m. Coils were constructed using copper wire, 0.79 mm in diameter; 250 turns were wound around the edge of a ½-inch plywood board having a radius of 64.3 cm, resulting in coils with average radii of 65 cm. The coils were mounted in a wooden frame such that they were parallel and separated by 65 cm.

The output of a signal generator (Wavetek Model 182A), was amplified (Krohn-Hite Model 7500) and fed to the coils. Performance of the system at 0. 1 mT, 60 Hz, was verified by direct measurement of the magnetic field (Magnetek AC Model 1846 flux probe) (Table 2).

DISCUSSION

The results of the analysis permit design of Helmholtz coils capable of providing uniform magnetic fields in any desired volume at any desired field strength. Choice of the volume within which the magnetic field has a specific degree of uniformity determines the minimum coil radius. The regions of uniformity have an inherently cylindrical symmetry (Figures 6 and 7), but there are points outside the cylinder for which the field is also within the specified variation. For variations ≥5%, the region of uniformity includes all values of z (Figure 7A). Almost all of the intercoil region of Helmholtz coils is uniform to within 50% (Figure 7D).

ACKNOWLEDGEMENT

This work was supported by the Memorial Medical Trust, Buffalo, New York.

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